In Figs. 1–4, the solid line represents the present numerical results from Eq. (22) and the dashed line is from the correlation Eq. (23). It can be seen that the present numerical results provide good agreement with the two limiting cases of pure forced[9] and pure free convection.[10,11] Thus, we can be confident of the accuracy of the present numerical scheme. It is noted that the maximum error between the present numerical results and the correlation equation is less than 8%.

Acknowledgment

The author acknowledges the National Science Council of the Republic of China for the financial support of this work through project NSC-79-0401-E006-28.

References

[1]Skelland, A. H. P., *Non-Newtonian Flow and Heat Transfer*, Wiley, New York, 1967.

[2]Metzner, A. B., "Heat Transfer in Non-Newtonian Fluids," *Advances in Heat Transfer*, Vol. 2, Academic Press, New York, 1965, pp. 389–392.

[3]Shenoy, A. V., and Mashelker, R. A., "Thermal Convection in Non-Newtonian Fluids," *Advances in Heat Transfer*, Vol. 15, Academic Press, New York, 1982, pp. 143–225.

[4]Irvine, T. F., Jr., and Karni, J., "Non-Newtonian Fluid Flow and Heat Transfer," *Handbook of Single-Phase Convective Heat Transfer*, Chap. 20, Wiley, New York, 1987.

[5]Bejan, A., *Convection Heat Transfer*, Wiley, New York, 1984.

[6]Lee, S. L., Chen, T. S., and Armaly, B. F., "New Finite Difference Solution Methods for Wave Instability Problems," *Numerical Heat Transfer*, Vol. 10, No. 1, 1986, pp. 1–18.

[7]Huang, M. J., and Lin, B. L., "Numerical Analysis of Forced Convection Over a Wedge in Non-Newtonian Power-Law Fluids for Any Modified Prandtl Number," *Journal of the Chinese Society of Mechanical Engineers*, Vol. 12, No. 3, 1991, pp. 250–256.

[8]Shenoy, A. V., "A Correlating Equation for Combined Laminar Forced and Free Convection Heat Transfer to Power-Law Fluids," *American Institute of Chemcial Engineers Journal*, Vol. 26, 1980, pp. 505–507.

[9]Acrivos, A., Shah, M. J., and Peterson, E. E., "Momentum and Heat Transfer in Laminary Boundary-Layer Flows of Non-Newtonian Fluids Past External Surfaces," *American Institute of Chemical Engineers Journal*, Vol. 6, No. 2, 1960, pp. 312–317.

[10]Shenoy, A. V., and Ulbrecht, J. J., "Temperature Profiles for Laminar Natural Convection Flow of Dilute Polymer Solutions Past an Isothermal Vertical Flat Plate," *Chemical Engineering Communications*, Vol. 3, 1979, pp. 303–324.

[11]Som, G., and Chen, J. L. S., "Free Convection of Non-Newtonian Fluids over Non-Isothermal Two-Dimensional Bodies," *International Journal of Heat and Mass Transfer*, Vol. 27, No. 5, 1984, pp. 791–794.

View Factors for Perpendicular and Parallel Rectangular Plates

J. R. Ehlert* and T. F. Smith†
University of Iowa, Iowa City, Iowa 52242

Nomenclature

A = area
d = distance between surfaces
F_{1-2} = view factor between surfaces 1 and 2
G = integration function
s = distance
x = distance
y = distance
η = distance
θ = polar angle
ξ = distance

Subscripts
1 = surface 1
2 = surface 2

Introduction

ANALYSIS of radiant exchange between surfaces separated by a radiatively transparent medium is of importance in several applications, including spacecraft thermal control, electronic thermal control, and building thermal environments. One procedure for performing the analysis is based on the radiosity-irradiation method.[1,2] Fundamental to this method is the calculation of the view factors that describe the radiant exchange between two surfaces. The view factors depend only on geometry of the participating surfaces. Evaluation of view factors continues to be of interest for a wide range of geometries. One geometric configuration that appears frequently is the parallel and perpendicular arrangements of surfaces. Howell[3] presented view factor expressions based on the analysis of Gross et al.[4] for rectangular perpendicular and parallel plates having parallel boundaries. During the use of these view factor expressions for developing a code to verify a three-dimensional discrete-ordinates model based on that of Sánchez and Smith,[5] it was apparent that simplified expressions could be developed. The purpose of this note is to present alternative expressions that are simplified over those available in the literature.

Analysis

Schematic diagrams of perpendicular and parallel surfaces labeled A_1 and A_2 are displayed in Fig. 1. It is of interest to obtain the view factor F_{1-2}. View factor algebra[1,2] could be used to find the required view factors but becomes quite lengthy and tedious as the number of surfaces increases and is subject to errors. The approach of Gross[4] is used to eliminate the need to perform view factor algebra. A notation similar to that of Gross et al.[4] is adopted for developing the view factor expressions.

The view factor is defined by

$$A_1F_{1-2} = \int_{A2}\int_{A1} \frac{\cos\theta_1 \cos\theta_2}{\pi s^2}\, dA_1\, dA_2 \qquad (1)$$

where θ_1 and θ_2 are polar angles for surfaces 1 and 2, respectively. Surface area A_1 has Cartesian coordinates x, y, and surface area A_2 has Cartesian coordinates η, ξ as shown in Fig. 1.

The distance and polar angles for the perpendicular plates are

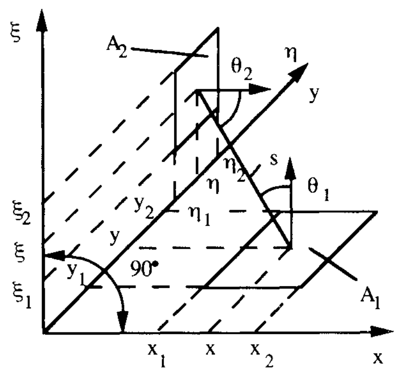
$$s^2 = x^2 + (y-\eta)^2 + \xi^2 \qquad \cos\theta_1 = \xi/s \qquad \cos\theta_2 = x/s \qquad (2)$$

which, when inserted into Eq. (1), give

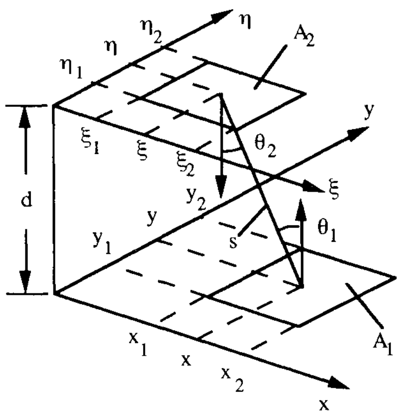
$$G(x, y, \eta, \xi) = \frac{1}{\pi}\int_\xi\int_\eta\int_y\int_x \cdot \frac{x\xi}{[x^2+\xi^2+(y-\eta)^2]^2}\, dx\, dy\, d\eta\, d\xi \qquad (3)$$

Received Feb. 13, 1992; revision received March 9, 1992; accepted for publication March 17, 1992.

*Graduate Student, Department of Mechanical Engineering.
†Professor, Department of Mechanical Engineering. AIAA Member.



a) Perpendicular plates



b) Parallel plates

Fig. 1 Plate arrangements.

The four integrations are performed using symbolic mathematics[6] to yield

$$G = \frac{1}{2\pi}\left\{(y-\eta)(x^2+\xi^2)^{1/2}\tan^{-1}\left[\frac{y-\eta}{(x^2+\xi^2)^{1/2}}\right] - \frac{1}{4}\right.$$
$$\left.\cdot[x^2+\xi^2-(y-\eta)^2]\ \ln\ [x^2+\xi^2+(y-\eta)^2]\right\} \tag{4}$$

The integration is performed in the order of x, ξ, y, η. Differentiation of Eq. (4) using symbolic mathematics[6] produced the integrand in Eq. (3). Only terms containing the four independent variables need to be retained. A series of additions is used to numerically carry out the evaluation of Eq. (4) using

$$A_1F_{1-2} = \sum_{l=1}^{2}\sum_{k=1}^{2}\sum_{j=1}^{2}\sum_{i=1}^{2}[(-1)^{(i+j+k+l)}G(x_i, y_j, \eta_k, \xi_l)] \tag{5}$$

The expression in Eq. (5) is the same as that found by Gross[4] and differs from that reported by Howell.[3] If all terms not having each of the four independent variables in them are eliminated in the expression reported by Howell,[3] then the expressions are the same. These terms are eliminated due to the canceling of these terms in Eq. (5) and, therefore, evaluation of those terms is computationally inefficient.

Parallel plate view factors are found using the same procedure as the perpendicular plate view factors. Figure 1b shows the needed arrangement of the surfaces. The geometric factors are

$$s^2 = d^2 + (x-\xi)^2 + (y-\eta)^2 \qquad \cos\theta_1 = \cos\theta_2 = d/s \tag{6}$$

Insertion of Eq. (7) into Eq. (1) gives

$$G(x, y, \eta, \xi) = \frac{d^2}{\pi}\int_\xi\int_\eta\int_y\int_x \cdot\frac{x\xi}{[d^2+(x-\xi)^2+(y-\eta)^2]^2}\,dx\,dy\,d\eta\,d\xi \tag{7}$$

The integration can be performed using symbolic mathematics[6] for the integration order of x, y, η, ξ to obtain

$$G = \frac{1}{2\pi}\Bigg((y-\eta)[(x-\xi)^2+d^2]^{1/2}\tan^{-1}$$
$$\cdot\left\{\frac{y-\eta}{[(x-\xi)^2+d^2]^{1/2}}\right\} + (x-\xi)[(y-\eta)^2+d^2]^{1/2}$$
$$\cdot\tan^{-1}\left\{\frac{x-\xi}{[(y-\eta)^2+d^2]^{1/2}}\right\} - \left\{\frac{d^2}{2}\ \ln\right.$$
$$\left.\cdot[(x-\xi)^2+(y-\eta)^2+d^2]\right\}\Bigg) \tag{8}$$

Equation (8) differs from the expression reported by Gross et al.[4] and Howell[3] as all terms not having all four of the independent variables are elminated due to the mathematical evaluation in Eq. (5). The possibility of the natural log expression becoming undefined is eliminated, unlike that for the previous expressions.

Discussions

Each of the view factor expressions given by the authors Howell[3] and Gross et al.[4] were evaluated for several arrangements of the surfaces. It was found necessary to set the variables to a very small number (10^{-40}) if the variable was equal to zero to prevent a division by zero error. Results for the view factors were found to be identical for both the perpendicular and parallel plates out to at least the ninth decimal digits depending on the geometry of the problem. The difference was due to computational rounding of the expressions.

The accuracy of the current expressions were further verified by checking the various problems in Chapman[7] and using view factor algebra. In all examples, agreement was found between view factors based on the current expressions and those obtained from view factor algebra. Using problem 9.42 as an example, the view factor as cited by Chapman[7] is 0.01556, and that given by Eqs. (4) and (5) is 0.015556. Finally, a radiosity-irradiation analysis for a rectangular enclosure where the enclosure walls are subdivided into a number of subsurfaces was developed to verify the enclosure and reciprocity relations of the view factors evaluated from the current expressions; the required relations were satisfied.

The reported expressions for computing the view factors for rectangular perpendicular and parallel plates with varying position and size having parallel boundaries satisfy the properties of the view factors.

Acknowledgments

J. R. Ehlert acknowledges the NASA Iowa Space Grant Consortium Summer Undergraduate Research Experience Program for financial support. T. F. Smith acknowledges support from the Rockwell International Corporation. Both authors express their thanks to A. L. Crosbie for inspiration to solve the multiple view factor integrals.

References

[1]Sparrow, E. M., and Cess, R. D., *Radiation Heat Transfer, Augmented Edition*, Hemisphere, Washington, DC, 1978.

[2]Siegel, R., and Howell, J. R., *Thermal Radiation Heat Transfer*,

Hemisphere, Washington, DC, 1981.
[3]Howell, J. R., *A Catalog of Radiation Configuration Factors*, McGraw-Hill, New York, 1982.
[4]Gross, U., Spindler, K., and Hahne, E., "Shapefactor-Equations for Radiation Heat Transfer Between Plane Rectangular Surfaces of Arbitrary Position and Size with Parallel Boundaries," *Letters in Heat and Mass Transfer*, Vol. 8, No. 3, 1981, pp. 219–227.
[5]Sanchez, A., and Smith, T. F., "Surface Radiation Exchange for Two-Dimensional Rectangular Enclosures Using the Discrete-Ordinates Method," *Journal of Heat Transfer*, Vol. 114, May 1992, pp. 465–472.
[6]Wolfram, S., *Mathematica*®: *A System for Doing Mathematics by Computer*, Addison-Wesley, New York, 1988.
[7]Chapman, A. J., *Fundamentals of Heat Transfer*, Macmillan, New York, 1987, p. 549.

Adaptive Grid Generation for the Calculation of Radiative Configuration Factors

C. J. Saltiel* and J. Kolibal†
University of Florida, Gainesville, Florida 32611

Introduction

MANY practical thermal and optical engineering applications require the evaluation of radiative configuration factors. For a number of simple geometries, closed-form expressions for configuration factors are well-cataloged[1]; however, most require some degree of numerical computation, especially if detailed surface properties are to be included in the radiative calculations. In many applications, such as combined-mode heat transfer problems, efficient and accurate numerical calculation schemes are essential. In moving boundary and time-dependent problems[2] (where exchange factors need to be calculated repeatedly), radiative calculations can be extremely computationally intensive. A comparative study of available methods for view-factor computation is found in Ref. 3.

Three methods for reducing computational error have been developed: 1) the *h*-method, involving refinement of the computational grid; 2) the *p*-method, in which the order of the approximation is increased; and 3) the *r*-method, in which nodal positions are relocated. Furthermore, these methods can be applied adaptively. The key to making adaption work effectively is in choosing robust criteria. As noted in Ref. 4, adaptive methods can increase the computational work if incorrectly applied.

Mesh refinement methods have been successfully employed in solving radiative and optical illumination problems.[3,5–7] Both adaptive *h*- and *p*-methods can be readily applied by evaluating local criteria, thus lending themselves to local adaption. The *r*-method, however, involves a global analysis. Relocating nodal positions may prove very valuable for sudden changes in the integrand, such as when blocking occurs (e.g., where a refined mesh might attempt to fit the nonshaded regions); however, redefining nodal points in more than one dimension can be algorithmically difficult.

In this note we demonstrate the advantages of adaptive *h*- and *p*-methods adaptively applied to evaluating radiative configuration factors by way of direct integration, using the finite element approach of Chung and Kim.[8] In particular, the adaptive criteria are modified as the solution of the problem proceeds in order to balance the requirements of efficiency and accuracy. Examples are drawn from simple geometries which clearly illustrate the implementation of the method for singularities associated with intersecting surfaces.

Received Aug. 23, 1991; revision received Feb. 10, 1992; accepted for publication Feb. 13, 1992.

*Assistant Professor, Department of Mechanical Engineering.
†Visiting Assistant Professor, Department of Mechanical Engineering.

View-Factor Formulation and Discretization

The general formula for the radiation configuration factor from a diffuse surface A to a diffuse surface B is expressible in terms of the angles θ_A and θ_B between the normals to the surfaces and the line connecting points r_A on surface A and r_B on surface B:

$$F_{A-B} = \frac{1}{A}\int_A\int_B \frac{\beta_{AB}\cos\theta_A\cos\theta_B}{\pi|r_A - r_B|^2}\,\mathrm{d}A\,\mathrm{d}B \tag{1}$$

In Eq. (1) β_{AB} is a blockage factor (zero if the line-of-sight is obstructed, and unity otherwise). While the value of the integrand in Eq. (1) is affected by the orientation of the surfaces (which decreases the integrand as sections along the surfaces become more nearly orthogonal), it is the $1/r^2$ singularity in the integrand which dominates the evaluation of the integral for connected or closely spaced surfaces.

Subdividing surfaces A and B into $(A_i)_{i=1}^M$ and $(B_j)_{j=1}^N$, the view-factor expression in Eq. (1) is discretized as

$$F_{A-B} = \frac{1}{A}\sum_{i=1}^{M}\sum_{j=1}^{N}\beta_{A_i-B_j}F_{A_i-B_j} \tag{2}$$

requiring integration for each element to element exchange:

$$F_{A_i-B_j} = \int_{A_i}\int_{B_j}\frac{\cos\theta_{A_i}\cos\theta_{B_j}}{\pi|r_{A_i} - r_{B_j}|^2}\,\mathrm{d}B_j\,\mathrm{d}A_i \tag{3}$$

Following Chung and Kim,[8] evaluation of Eq. (3) uses a mapping of local to global coordinates by means of the Jacobian J of the transformation, i.e.

$$\mathrm{d}A_i = \mathrm{d}x_{A_i}\,\mathrm{d}y_{A_i} = |J|_{A_i}\,\mathrm{d}\xi_{A_i}\eta_{A_i} \tag{4}$$

$$\mathrm{d}B_j = \mathrm{d}x_{B_j}\,\mathrm{d}y_{B_j} = |J|_{B_j}\,\mathrm{d}\xi_{B_j}\eta_{B_j} \tag{5}$$

As pointed out in Ref. 8, improvement in accuracy may be achieved by using higher-order finite-element interpolation functions, additional number of quadrature points, and/or more elements. The drawback to uniformly increasing quadrature order, points, or the number of elements is the computational cost.

Adaptive Criteria

Ideally, the evaluation of the integral Eq. (1) should depend on the functional form of the integrand. This is especially the case in complicated geometries where the integrand is often rapidly varying. We consider adaptive mechanisms for adequately sampling the contributions to the integral using grid refinement or local increase in the order of the quadrature based on an estimate of the local error. The objective of the adaptive scheme is to achieve a required accuracy while reducing the number of functional evaluations which grow as the product of the number of mesh points on all surfaces.

We construct the adaptive algorithm using three criteria as shown in Fig. 1. The first of these, C_1, uses the local relative error to determine whether an element should be split or its quadrature increased, and the second, C_2, evaluates the importance of the contribution of each element to the overall integral. A third criterion, C_3, is introduced to assess and modify the previous two criteria. While the rules for modification are elementary in this study, they lend themselves readily to rule-based methods in which geometries can be categorized.